

# General dynamical properties of cosmological models with nonminimal kinetic coupling

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## Abstract

© 2018 IOP Publishing Ltd and Sissa Medialab. We consider cosmological dynamics in the theory of gravity with the scalar field possessing the nonminimal kinetic coupling to curvature given as  $\eta G_{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ , where  $\eta$  is an arbitrary coupling parameter, and the scalar potential  $V(\phi)$  which assumed to be as general as possible. With an appropriate dimensionless parametrization we represent the field equations as an autonomous dynamical system which contains ultimately only one arbitrary function  $\chi(x) = 8\pi\eta V(x/8\pi)$  with  $x = 8\pi\phi$ . Then, assuming the rather general properties of  $\chi(x)$ , we analyze stationary points and their stability, as well as all possible asymptotical regimes of the dynamical system. It has been shown that for a broad class of  $\chi(x)$  there exist attractors representing three accelerated regimes of the Universe evolution, including de Sitter expansion (or late-time inflation), the Little Rip scenario, and the Big Rip scenario. As the specific examples, we consider a power-law potential  $V(\phi) = M^4(\phi/\phi_0)^\sigma$ , Higgs-like potential  $V(\phi) = \lambda/4(\phi^2 - \phi_0^2)^2$ , and exponential potential  $V(\phi) = M^4 e^{-\phi/\phi_0}$ .

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## Keywords

dark energy theory, inflation, modified gravity

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